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Monodromy calculations of fourth order equations of Calabi-Yau type. (English summary)
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In this article, the authors study the monodromies of fourth-order ODEs from the viewpoint of homological mirror symmetry.

Let X be a Calabi-Yau threefold with Picard number one. Assume that X has a one-parameter mirror family $\mathcal{Y}\to\mathbb{P}^1$ of Calabi-Yau threefolds. Then the Picard-Fuchs operator L,

$$L=a_4(z)\frac{d^4}{dz^4}+a_3(z)\frac{d^3}{dz^3}+a_2(z)\frac{d^2}{dz^2}+a_1(z)\frac{d}{dz}+a_0(z),$$

describing the variation of the Hodge structure of $\mathcal{Y}$ is fourth order and must satisfy several properties. Among them is that L has a singular point with maximal unipotent monodromy, so that the mirror map is constructed from solutions around that point. Another condition is that the genus zero instanton numbers of X computed from the Yukawa coupling are integers. (See Appendix B.)

Fourth-order ODEs satisfying the above properties are collected in [G. Almkvist and V. V. Zudilin, in *Mirror symmetry. V*, 481–515, Amer. Math. Soc., Providence, RI, 2006; MR2282972 (2008j:14073)] and [G. Almkvist, C. van Enckevort, D. van Straten and W. Zudilin,"Tables of Calabi-Yau equations", preprint, arxiv.org/abs/math/0507430]. A motivation of this paper is to list ODEs that do have a geometric origin (i.e., X). For this purpose, the authors use as their guide Kontsevich's homological mirror symmetry conjecture [M. Kontsevich, in *Proceedings of the International Congress of Mathematicians, Vol. 1, 2 (Zürich, 1994)*, 120–139, Birkhäuser, Basel, 1995; MR1403918 (97f:32040)]. Based on Kontsevich's observation in the case in which X is a quintic hypersurface in $\mathbb{P}^4$, they further require an ODE to satisfy the following conditions: 1. there exists a singular point with spectrum $(0,1,1,2)$, called the conifold point; 2. the monodromy matrices $T_{\mathrm{DM}}, S_{\mathrm{DM}}$ at the maximal unipotent monodromy point and the conifold point are of the form

$$T_{\mathrm{DM}}=\begin{pmatrix}1&1&0&0\\0&1&d&0\\0&0&1&1\\0&0&0&1\end{pmatrix},\quad S_{\mathrm{DM}}=\begin{pmatrix}1&0&0&0\\-k&1&0&0\\-1&0&1&0\\-1&0&0&1\end{pmatrix}$$

with respect to some basis of $H_3(Y,\mathbb{Q})$ where $Y=\rho^{-1}(0)$.

If the ODE is assumed to be a Picard-Fuchs operator of a family $\mathcal{Y}$ mirror to a Calabi-Yau threefold X, then the number d must be equal to the triple intersection $d=H^3$ of the generator H of $H^2(X,\mathbb{Z})$ and k must be $k=\frac{c_2(X)\cdot H}{12}+\frac{H^3}{6}$. Moreover, the Euler number $c_3(X)$ can be computed from a conifold period, as explained in Section 5.

The authors compute monodromy matrices of ODEs in the lists in [G. Almkvist, C. van Enckevort, D. van Straten and W. Zudilin, op. cit.] and find 64 ODEs that satisfy the above conditions.

They also obtain the numbers $H^3, c_2(X) \cdot H, c_3(X)$. The result is summarized in Table 1. For ODEs whose geometric origins are known, the corresponding Calabi-Yau threefolds X are also listed: these include the 14 hypergeometric cases, the cases from complete intersection in Grassmannians, 7×7 Pfaffian Calabi-Yau in $\mathbb{P}^6$ [E. A. Rødland, Compositio Math. **122** (2000), no. 2, 135–149; MR1775415 (2001h:14051)] and Tjøtta's example [E. N. Tjøtta, Compositio Math. **126** (2001), no. 1, 79–89; MR1827863 (2002h:14094)]. They also conjecture X for some other ODEs. A few interesting examples are described in detail in Section 7.

{For the entire collection see MR2282005 (2007h:14002)}

Reviewed by *Yukiko Konishi*